

# Ground Motion Convolution/Deconvolution in Layered Half Space with an Accelerator Tunnel at depth

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# Problem Definition

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## ■ GOAL:

Given a potential site with a layered profile estimate the amplification and/or reduction of motion as it filters through the various layers such that the quietest of the layers can be assessed

## ■ Issues:

- sources of ground motion initiate at bedrock or at surface locations
- knowledge of soil layer configuration and properties – need for site exploration
- Use of engineering computational tools to assess site response

# What we know thus far

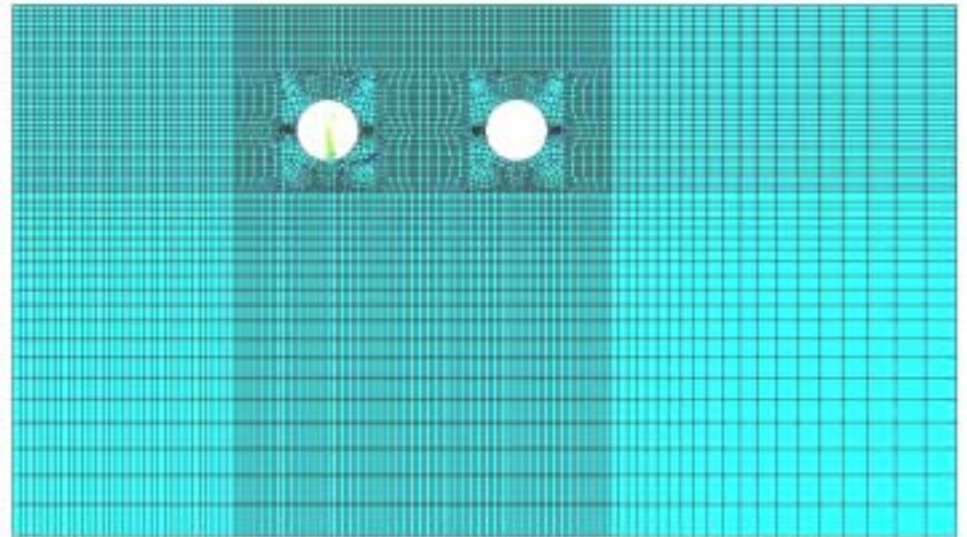
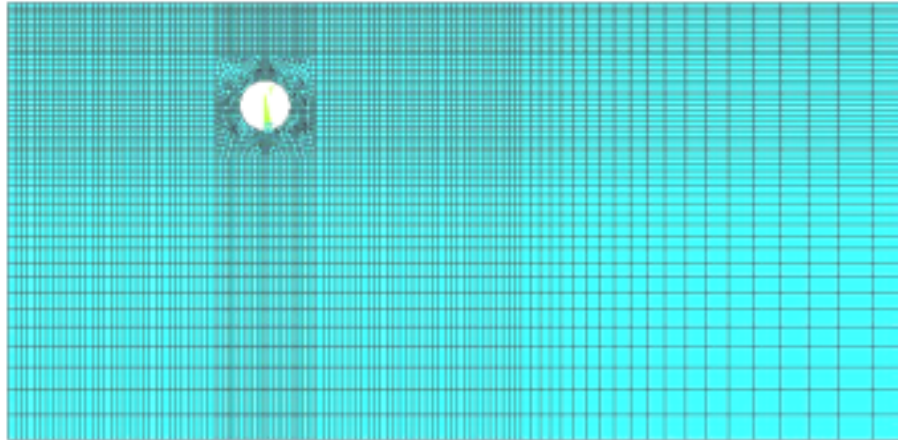
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- Stay away from the top layer !!!
- Have at least one soil interface above the tunnel
- It is better to have the accelerator tunnel within a single layer
- Avoid being close to interfaces to eliminate waves trapped in the small interface zone
- When on bedrock, have to deal with micro-seismic activity
  - ways to isolate the tunnel
  - Given site conditions it may be possible to find quieter location above the bedrock

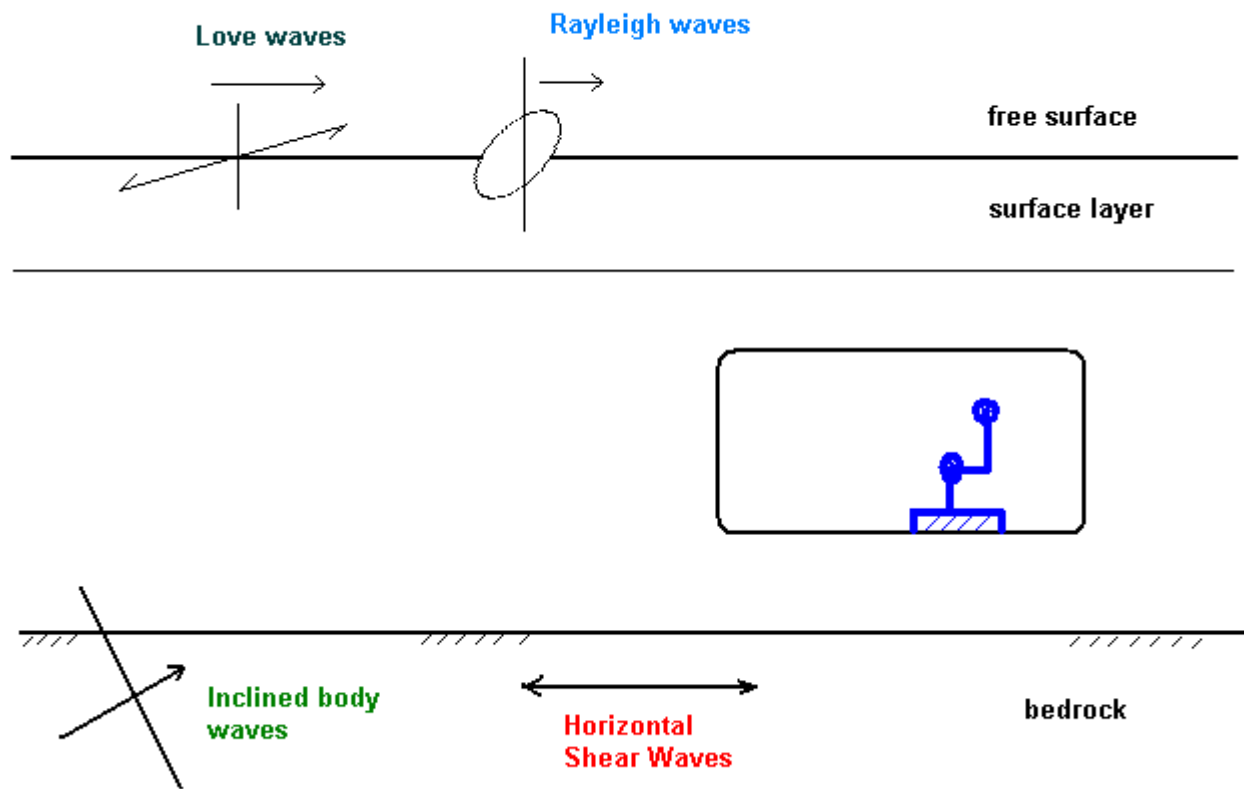
# Principles of Motion Convolution/De-convolution

## Accel. Tunnel - Soil Configuration

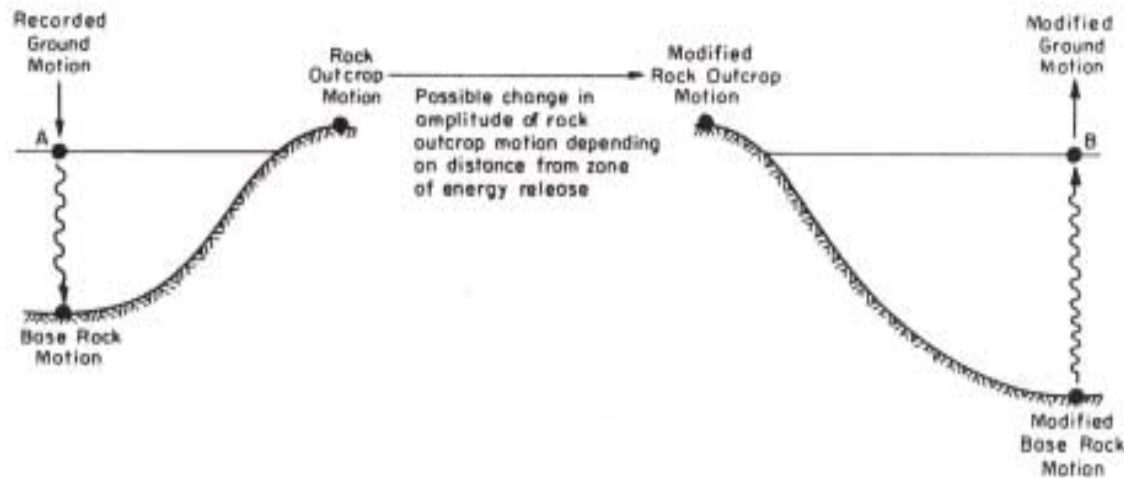
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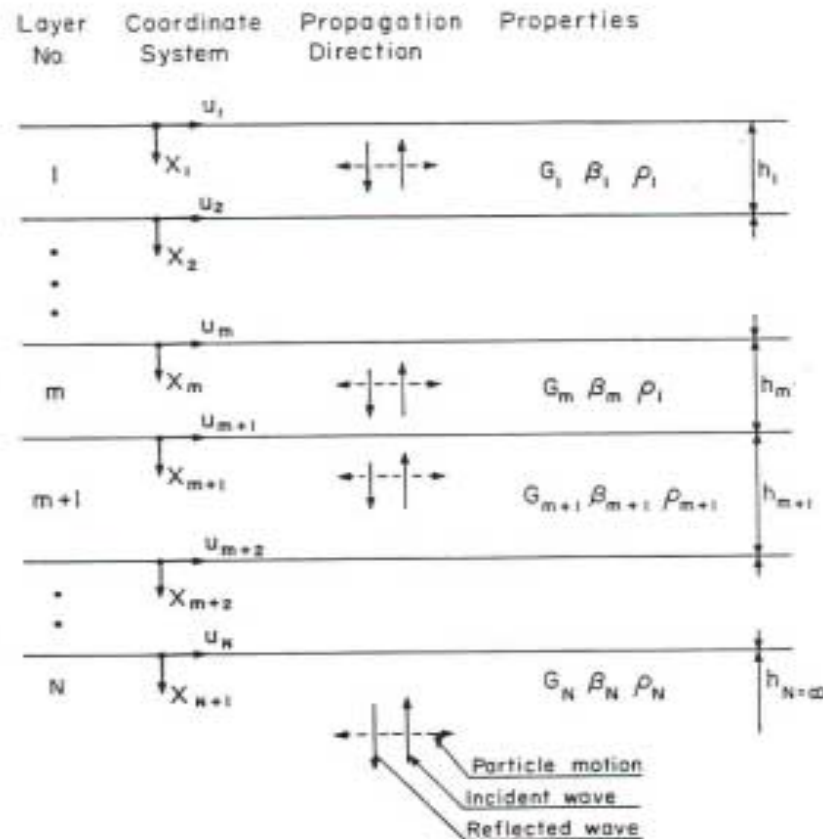
# Principles of Motion Convolution/De-convolution



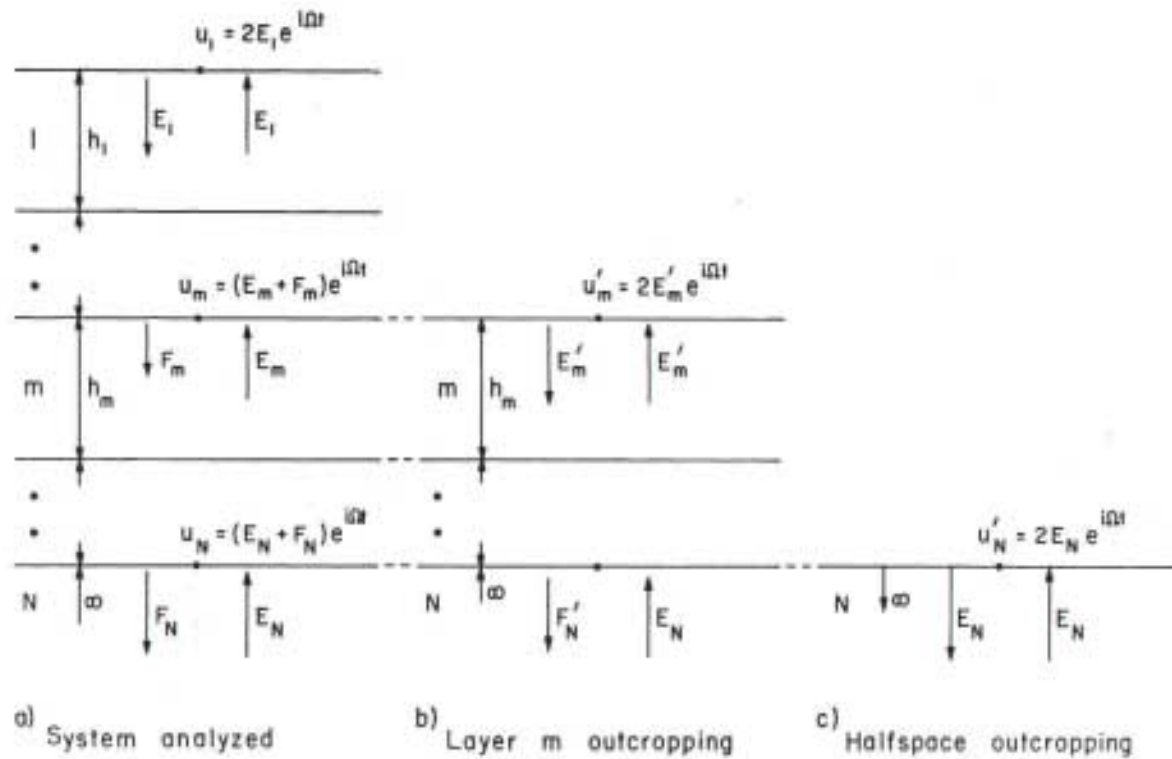
# Principles of Motion Convolution/De-convolution



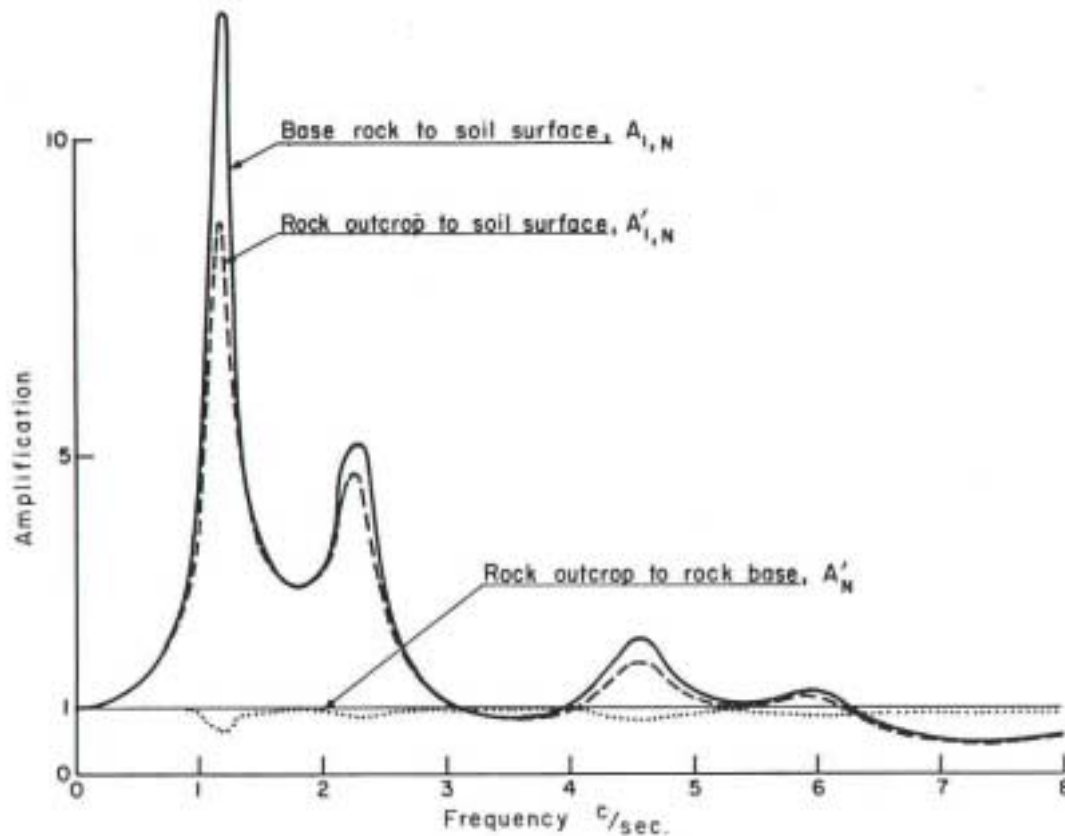
# Principles of Motion Convolution/De-convolution



# Principles of Motion Convolution/De-convolution



# Principles of Motion Convolution/De-convolution



# Engineering Computational Tools

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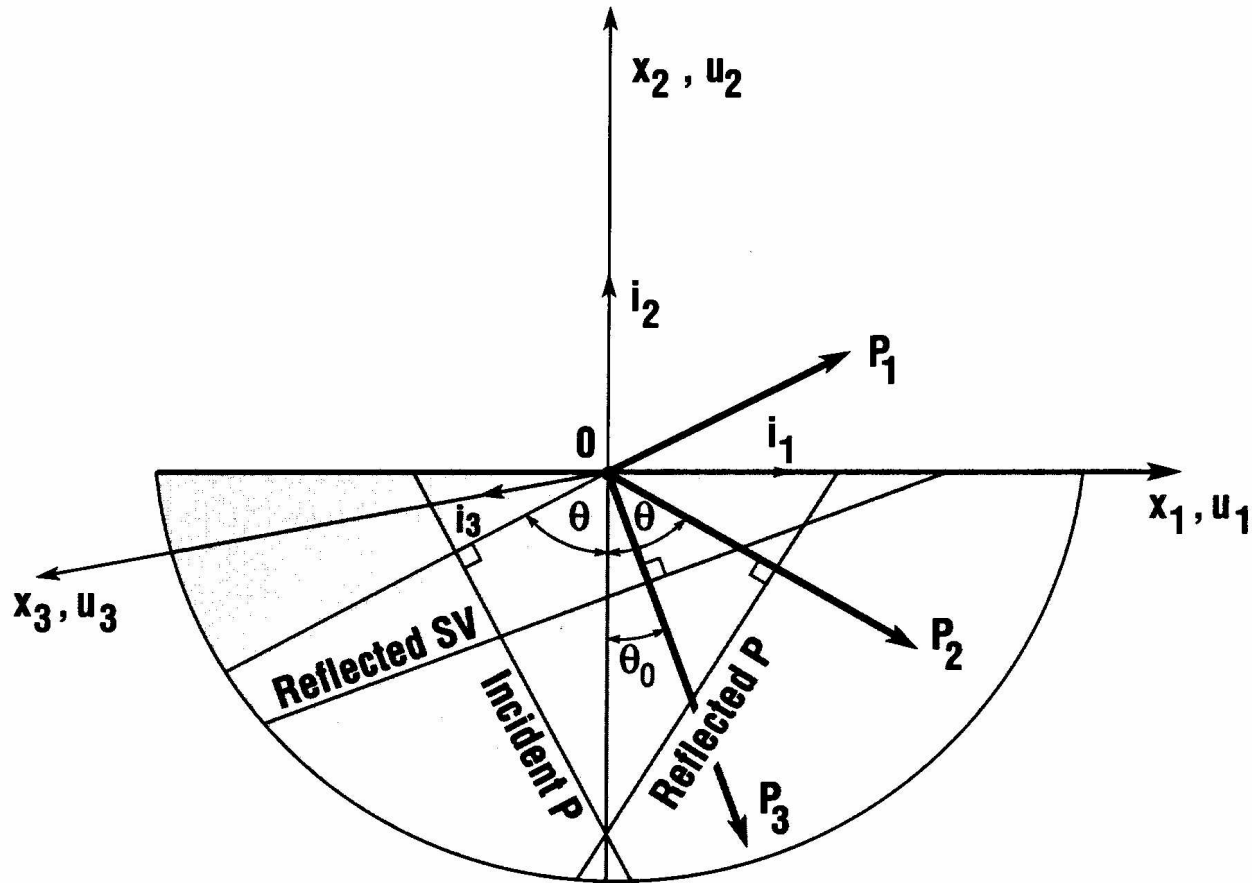
- **Site motion amplification**

- POROSLAM – SHAKE – MASH – CARES – SPECTRA - DIGES

- **Solve for:**

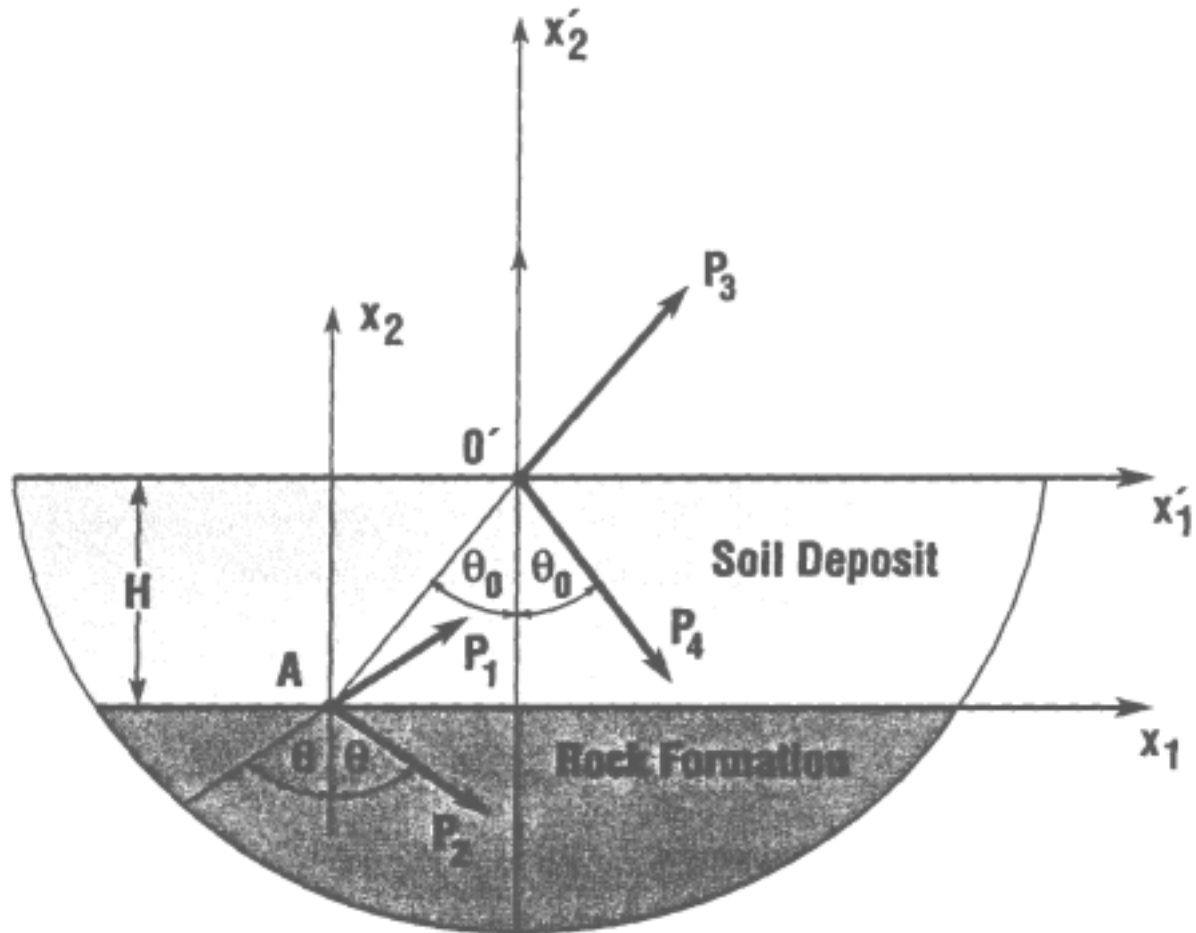
- Motion amplification – convolution/deconvolution
- Study the effect of key properties in a 1-D analysis (SHAKE, CARES)
- Incorporate soil saturation and water table in a multi-layered system that is close to the real site profile using the POROSLAM code (2-D frequency-based analysis). In the case of variation of the location of water table, its impact on motion filtering needs special attention

# Convolution - Deconvolution in Layered Halfspace



**Uniform Soil Deposit: P-Wave Incidence**

# Convolution - Deconvolution in Layered Halfspace



Soil Deposit Overlying Rock: SH-Waves

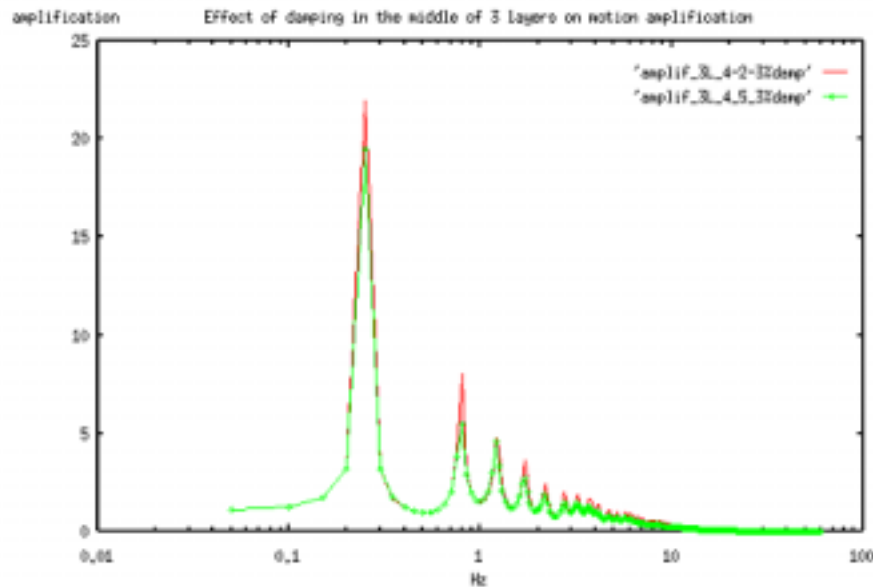
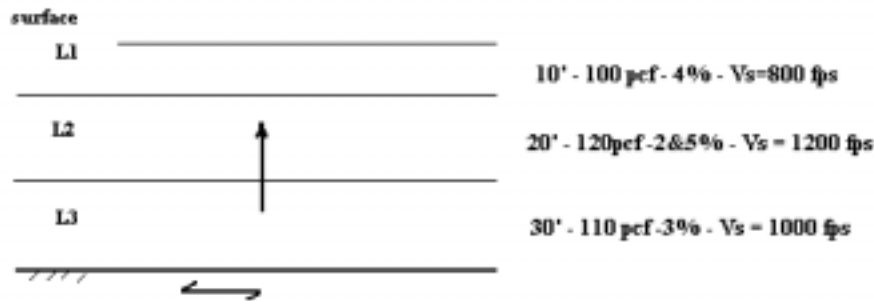
# Convolution - Deconvolution in Layered Halfspace

## CONVOLUTION - DECONVOLUTION PROCESS

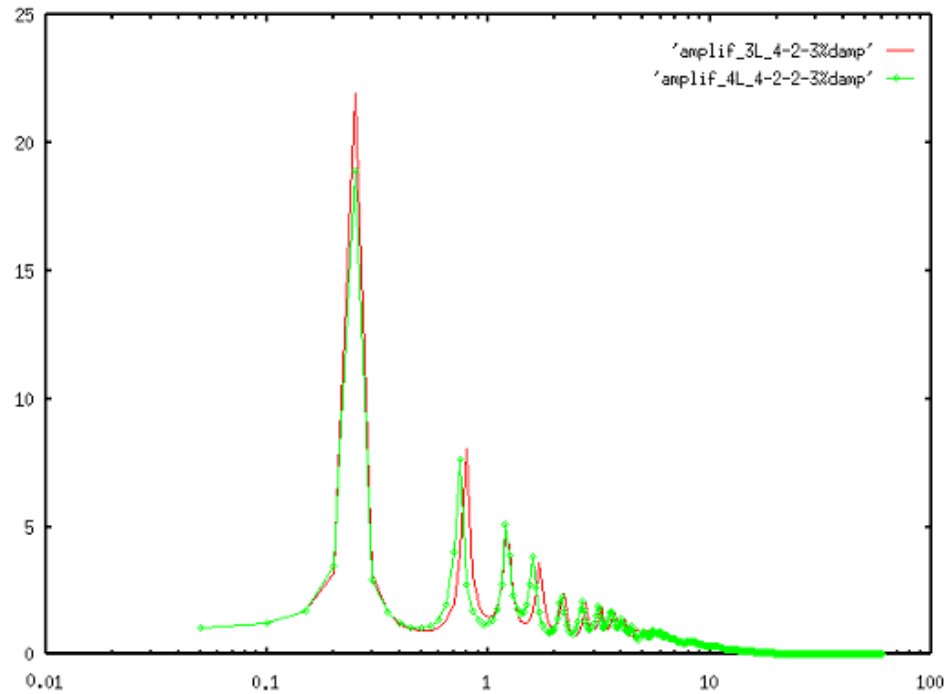
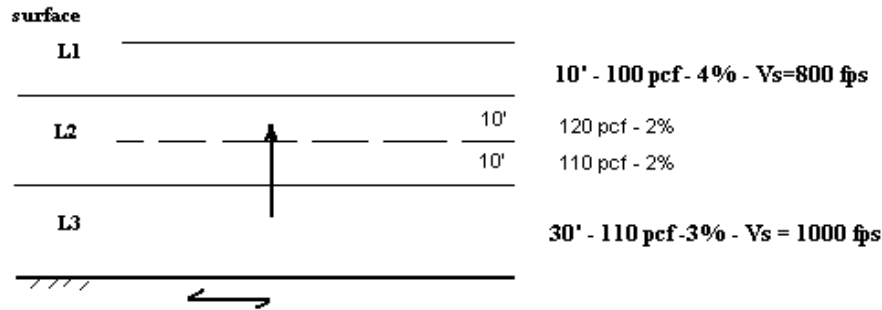
### Governing Principle of Motion Transfer

No rotational motion due to scattering

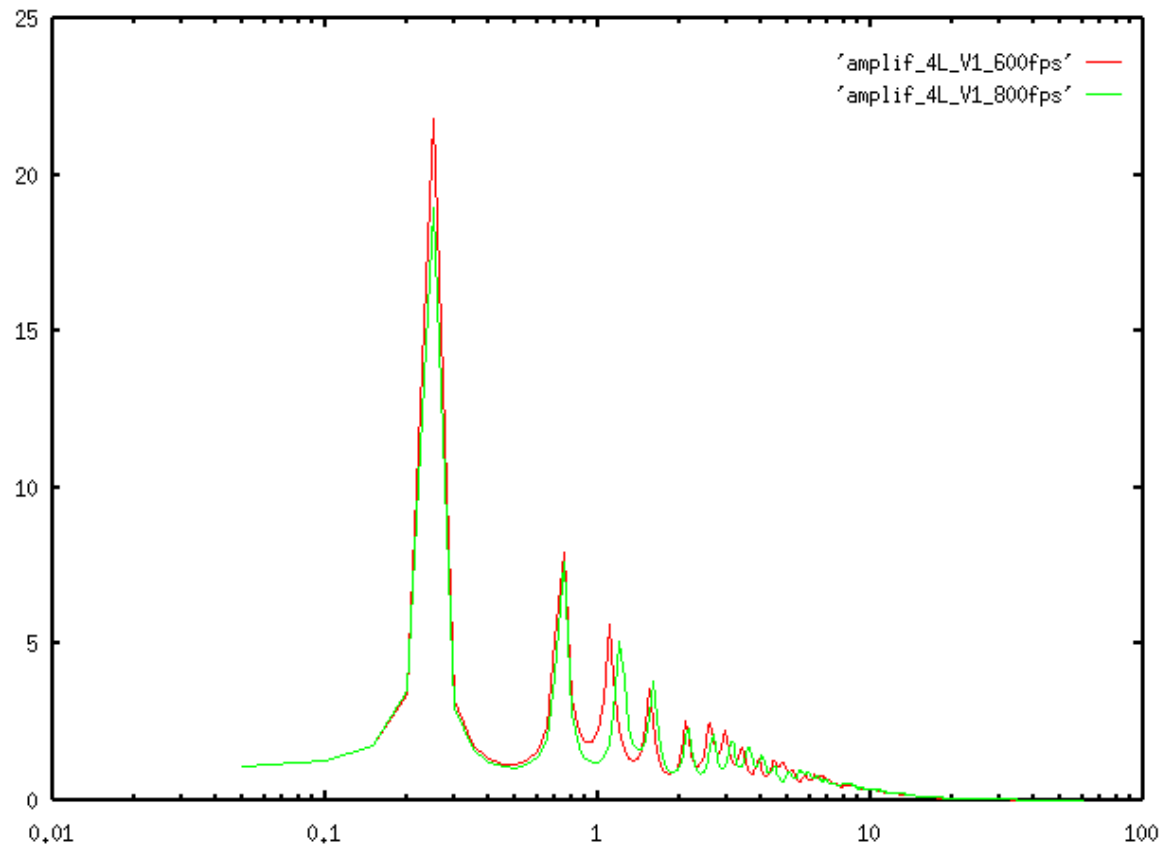
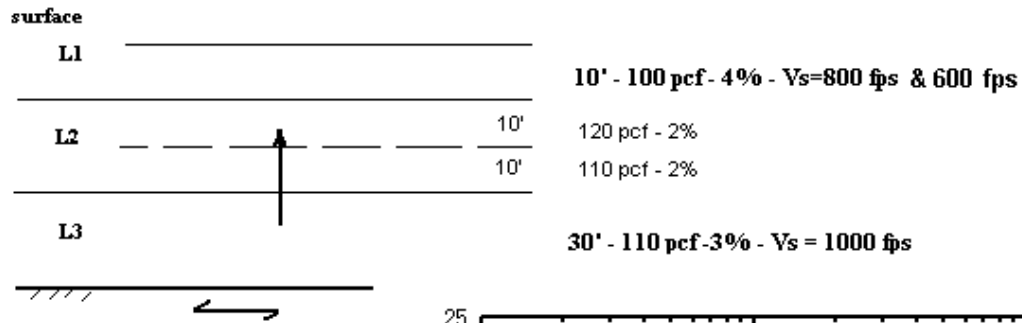
$$H(\omega) = \begin{bmatrix} [H_T(\omega)] \\ \text{---} \\ [0] \end{bmatrix}$$



# Convolution - Deconvolution in Layered Halfspace



# Convolution - Deconvolution in Layered Halfspace



# Convolution - Deconvolution in Layered Halfspace SH Waves

## Inclined SH Waves

**Displacement Vector**  $u(r, \omega, t) = Ae^{ik(r \cdot p - c_s t)} \hat{d}$

where

$p = \sin\theta \hat{i}_1 + \cos\theta \hat{i}_2 =$  propagation vector

$\hat{d} = \hat{i}_3 =$  direction of particle motion

$r = x_1 \hat{i}_1 + x_2 \hat{i}_2 =$  position vector

$A, k, c_s$  are the amplitude, wavenumber and phase velocity respectively

The requirement that the surface is free of traction yields that the reflected wave is in phase with the incident wave. The total displacement is then

$$u_3(x_1, x_2; \omega, t) = 2A \cos\alpha_\theta e^{ik(x_1 \sin\theta - c_s t)}$$

$$\alpha_\theta = \frac{\omega x_2}{c_s} \cos\theta$$

Thus, the transfer function between the displacement (or acceleration) at a depth  $x_2 = -h$  and its counterpart at the surface is expressed in the form

$$H(\omega) = \frac{u_3(x_1, -h; \omega, t)}{u_3(x_1, 0; \omega, t)} = \cos\alpha_h$$

$$\alpha_h = \frac{\omega h}{c_s} \cos\theta$$

# Convolution - Deconvolution in Layered Halfspace P- Waves

Incident **P-wave**:

$$[u_1 \ u_2 \ u_3]^T = A_1 [\sin\theta \ \cos\theta \ 0]^T e^{ik(r \cdot p_1 - c_p t)}$$

Reflected **P-wave**:

$$[u_1 \ u_2 \ u_3]^T = A_2 [\sin\theta \ -\cos\theta \ 0]^T e^{ik(r \cdot p_2 - c_p t)}$$

Reflected **SV-wave**:

$$[u_1 \ u_2 \ u_3]^T = A_3 [\cos\theta_0 \ \sin\theta_0 \ 0]^T e^{ik_0(r \cdot p_3 - c_s t)}$$

$c_p, c_s$  : **P** and **SV** wave velocity respectively

$A_1, A_2, A_3$  : Wave amplitudes

$p_1, p_2, p_3$  : propagation vectors of incident **P**, reflected **P** and reflected **SV**-wave

while,  $\sin\theta_0 = \frac{\sin\theta}{s}$   $k_0 = sk$  and  $s = c_p/c_s$

The amplitudes  $A_1, A_2$  and  $A_3$  satisfy the relation

$$\begin{bmatrix} \lambda + 2\mu\cos^2\theta & -s\mu\sin 2\theta \\ -\mu\sin 2\theta & -s\mu\cos 2\theta \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = - \begin{bmatrix} \lambda + 2\mu\cos^2\theta \\ \mu\sin 2\theta \end{bmatrix}$$

where  $q_1 = \frac{A_2}{A_1}$   $q_2 = \frac{A_3}{A_1}$  and  $\lambda, \mu$  are the Lamé constants of the halfspace.

# Convolution - Deconvolution in Layered Halfspace P - Waves

## TOTAL DISPLACEMENT due to INCIDENT & REFLECTED WAVES

Transfer Function between  
horizontal & vertical displ. at free  
surface

$$\frac{u_1|_{x_2=0}}{u_2|_{x_2=0}} = \frac{(1 + q_1) \sin\theta + q_2 \cos\theta_0}{(1 - q_1) \cos\theta + q_2 \sin\theta_0}$$

The transfer function between the vertical displacement at a depth  $h$  from the free surface and the vertical displacement at the surface is of the form

$$\frac{u_2|_{x_2=-h}}{u_2|_{x_2=0}} = \frac{(e^{-i a_L h} - q_1 e^{i a_L h}) \cos\theta + q_2 e^{i a_s h} \sin\theta_0}{(1 - q_1) \cos\theta + q_2 \sin\theta_0}$$

Similarly, the transfer function between the horizontal displacement at a depth  $h$  and the horizontal displacement at the surface takes the form

$$\frac{u_1|_{x_2=-h}}{u_1|_{x_2=0}} = \frac{(e^{-i a_L h} + q_1 e^{i a_L h}) \sin\theta + q_2 e^{i a_s h} \cos\theta_0}{(1 + q_1) \sin\theta + q_2 \cos\theta_0}$$

where  $a_L = \frac{\omega h}{c_L} \cos\theta$  and  $a_s = \frac{\omega h}{c_s} \cos\theta_0$  represent dimensionless frequencies for P and SV waves respectively.

Transfer Function for Vertical Incidence ( $\theta = 0$ )

Vertical displ at depth  $h$  vs. vertical displ at free surface

$$\frac{u_2|_{x_2=-h}}{u_2|_{x_2=0}} = \cos a_L^0 \quad a_L^0 = \frac{\omega h}{c_L}$$

# Convolution - Deconvolution in Layered Halfspace

## Inclined SH-wave in Soil Deposit Overlying a Rock Formation

Incident **SH**-wave:

$$u_3^{(1)} = A_1 e^{ik(r \cdot p_1 - c_{S,R}t)}$$

Reflected **SH** at interface:

$$u_3^{(2)} = A_2 e^{ik(r \cdot p_2 - c_{S,R}t)}$$

Refracted SH waves across interface

$$u_3^{(3)} = A_3 e^{ik(r' \cdot p_1 - c_{S,S}t)}$$

Reflected **SH**-wave at surface:

$$u_3^{(4)} = A_4 e^{ik_0(r' \cdot p_1 - c_{S,S}t)}$$

where,

$c_{S,R}, c_{S,S}$  : **SH**-wave velocities in the rock and soil respectively

$r, r'$  : position vectors WRT coordinate systems

$p_1, \dots, p_4$  : propagation vectors

$A_1, \dots, A_4$  : wave amplitudes

$k, k_0$  : wave numbers

$\mu_S, \mu_R$  : shear moduli of soil and rock respectively

# Convolution - Deconvolution in Layered Halfspace

## Transfer Function Generation

Continuity of total stresses  $\tau_{23}$  and total displacements  $u_3$  across the interface yields,

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \frac{A_3}{2} \begin{bmatrix} 1-q & 1+q \\ 1+q & 1-q \end{bmatrix} \begin{bmatrix} e^{i\alpha_H} \\ e^{-i\alpha_H} \end{bmatrix}$$

where,

$$q = \frac{c_{S,R} \cos \theta_{0R}}{c_{S,S} \cos \theta_{0S}} \alpha_H = \frac{\omega H}{c_{S,S}} \sin \theta_0 = \frac{c_{S,S}}{c_{S,R}} \sin \theta$$

Based on the above relations the following transfer functions can be deduced:

$$H_1(\omega) = \frac{\text{motion at depth } x_2' = -h}{\text{motion at surface}} = \frac{1}{2} (e^{i\alpha_H} + e^{-i\alpha_H})$$

$$H_2(\omega) = \frac{\text{motion at depth } x_2' = -h}{\text{motion at interface}} = \frac{e^{i\alpha_h} + e^{-i\alpha_h}}{e^{i\alpha_H} + e^{-i\alpha_H}}$$

$$H_3(\omega) = \frac{\text{motion at depth } x_2' = -h}{\text{at interface (no top soil)}} = \frac{e^{i\alpha_h} + e^{-i\alpha_h}}{(1-q)e^{i\alpha_H} + (1+q)e^{-i\alpha_H}}$$

# Convolution - Deconvolution in Layered Halfspace

## Kinematic Interaction

Transfer Function

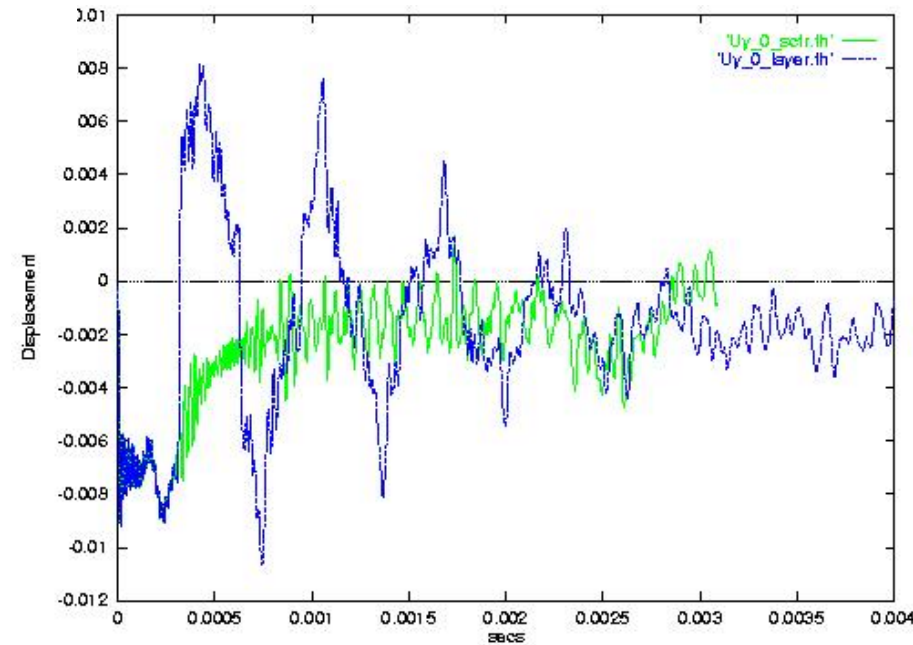
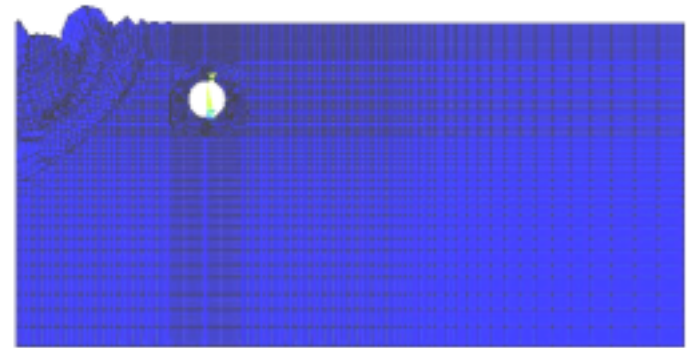
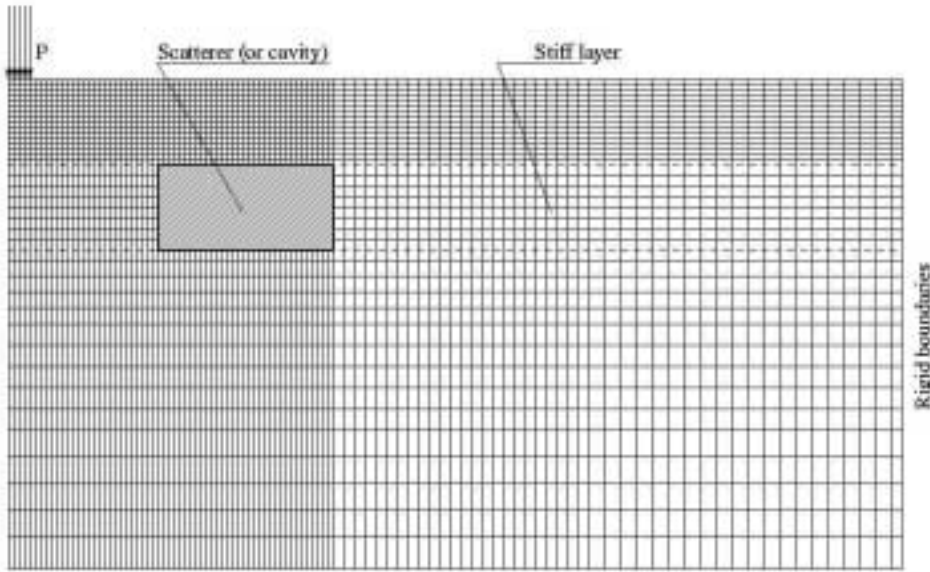
$$H(\omega) = \begin{bmatrix} [H_{transl}(\omega)] \\ \text{---} \\ [H_{rotnl}(\omega)] \end{bmatrix}$$

$H(\omega)$  is a [6 x 3]  
matrix

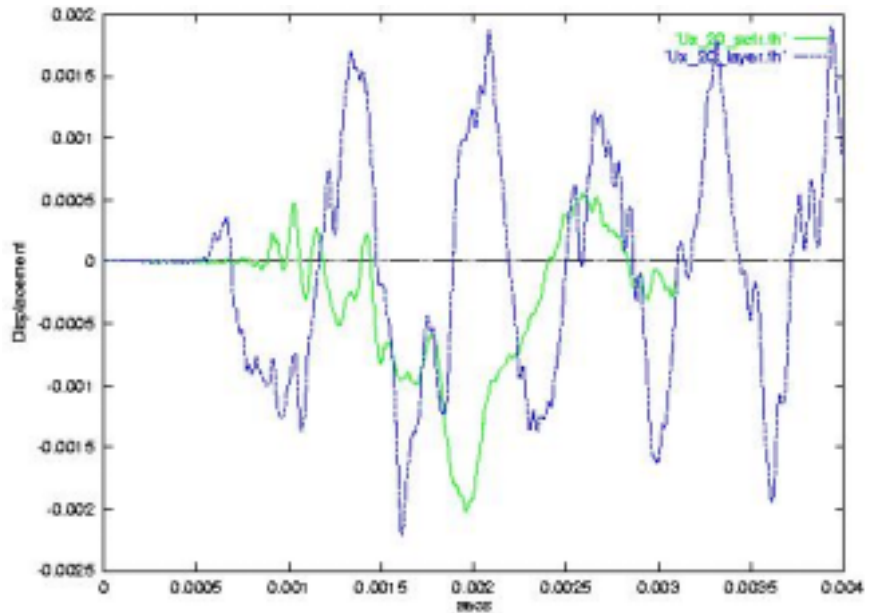
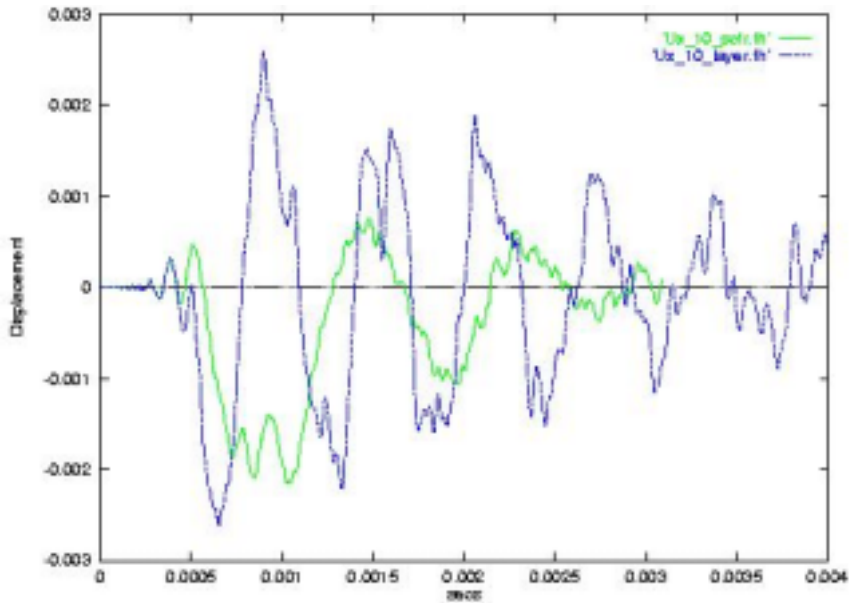
The dependency of the foundation input motion on the geometry of the massless foundation and the interface condition with the soil, on the properties and the stratigraphy of the underlying soil and on the nature of waves leads to a complex problem.

- o The **direct** application of the free-field motion on the foundation results in only translational components of the foundation motion regardless the soil stratigraphy, wave-type and the position of the foundation depthwise.
- o The **convolution/deconvolution** approach does incorporate the stratigraphy by including the outcrop motion, the wave-type and the position of the foundation mat but it fails to introduce any torsional components on the foundation.
- o The **kinematic interaction** approach incorporates the wave-type, the foundation geometry the soil deposit stratigraphy (analytic closed-form solutions are possible only for uniform halfspace) and allows the torsional and rocking effects to appear in the lower half of the scattering matrix.

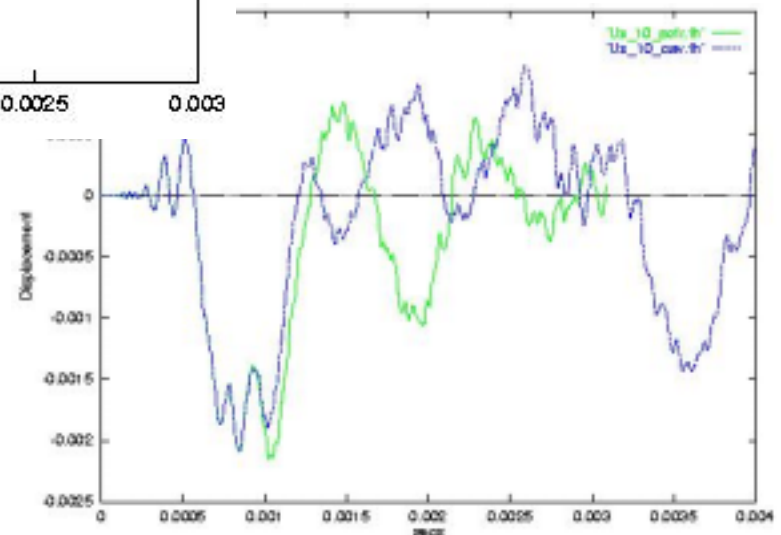
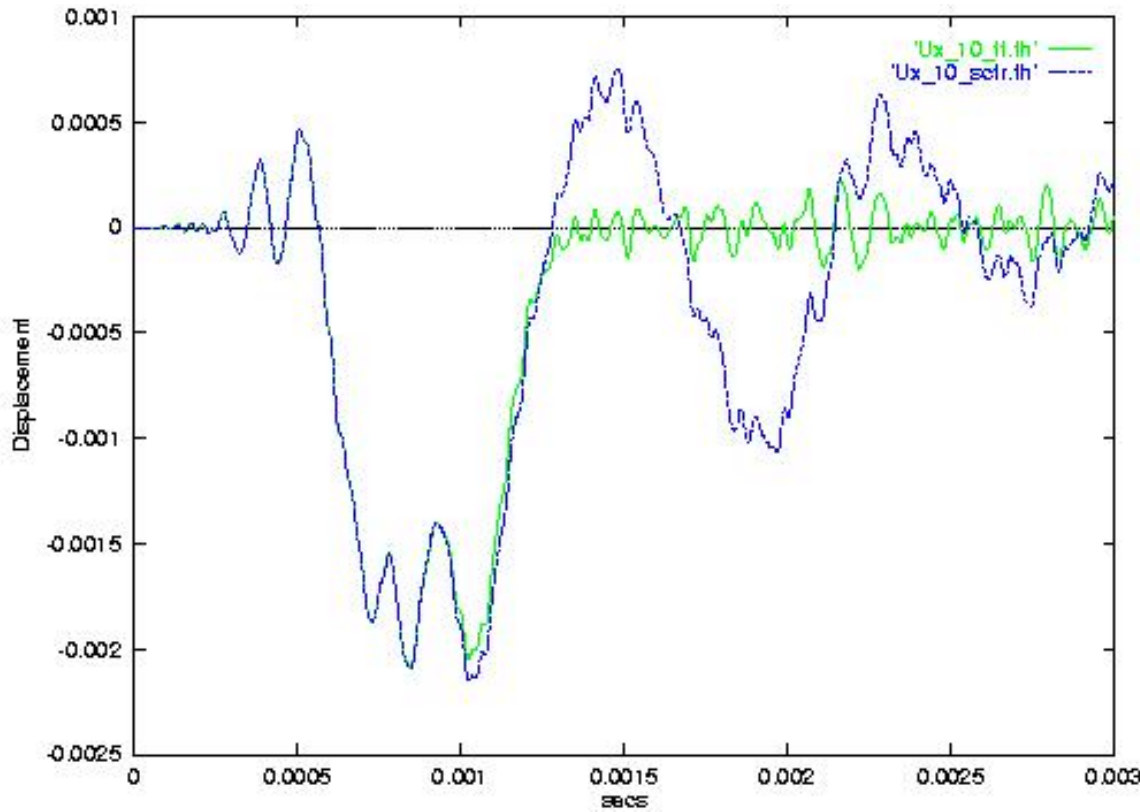
# Site layer effect on motion propagation



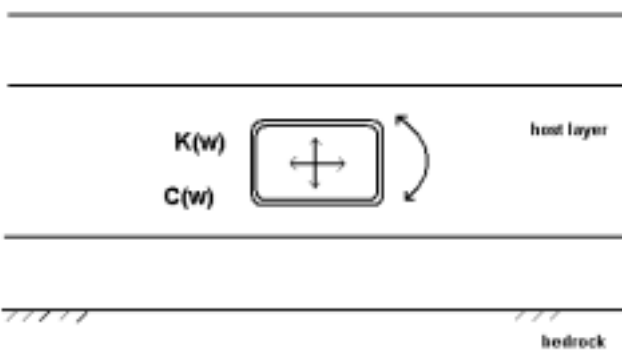
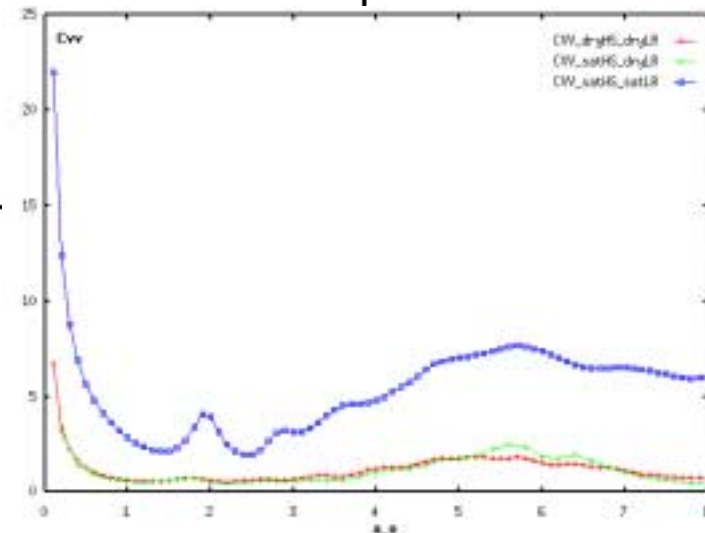
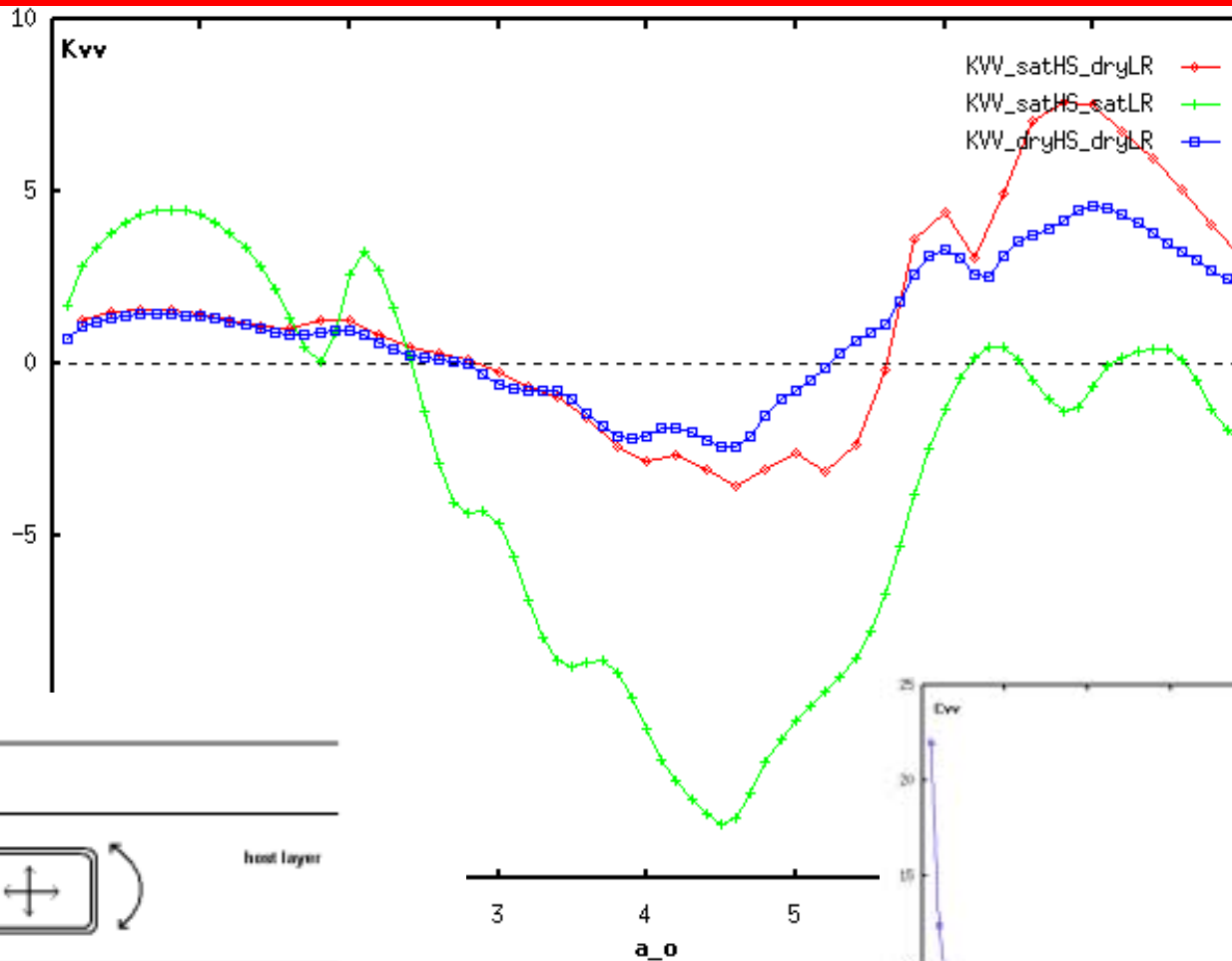
# Layer effect on motion propagation



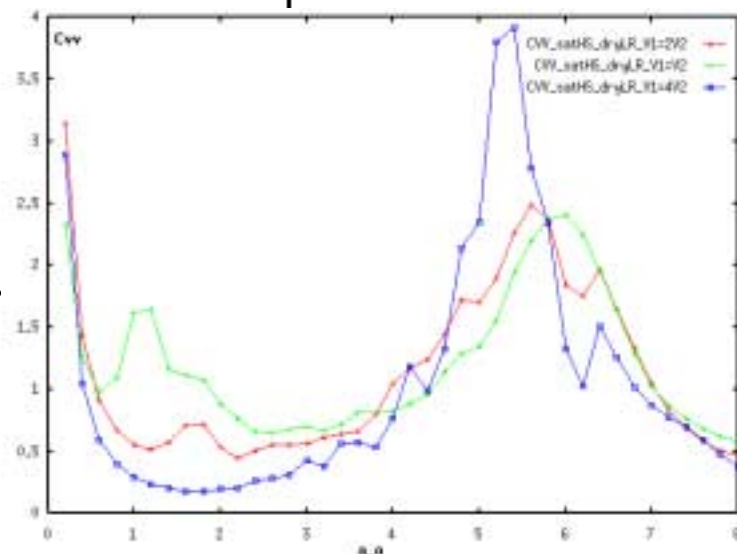
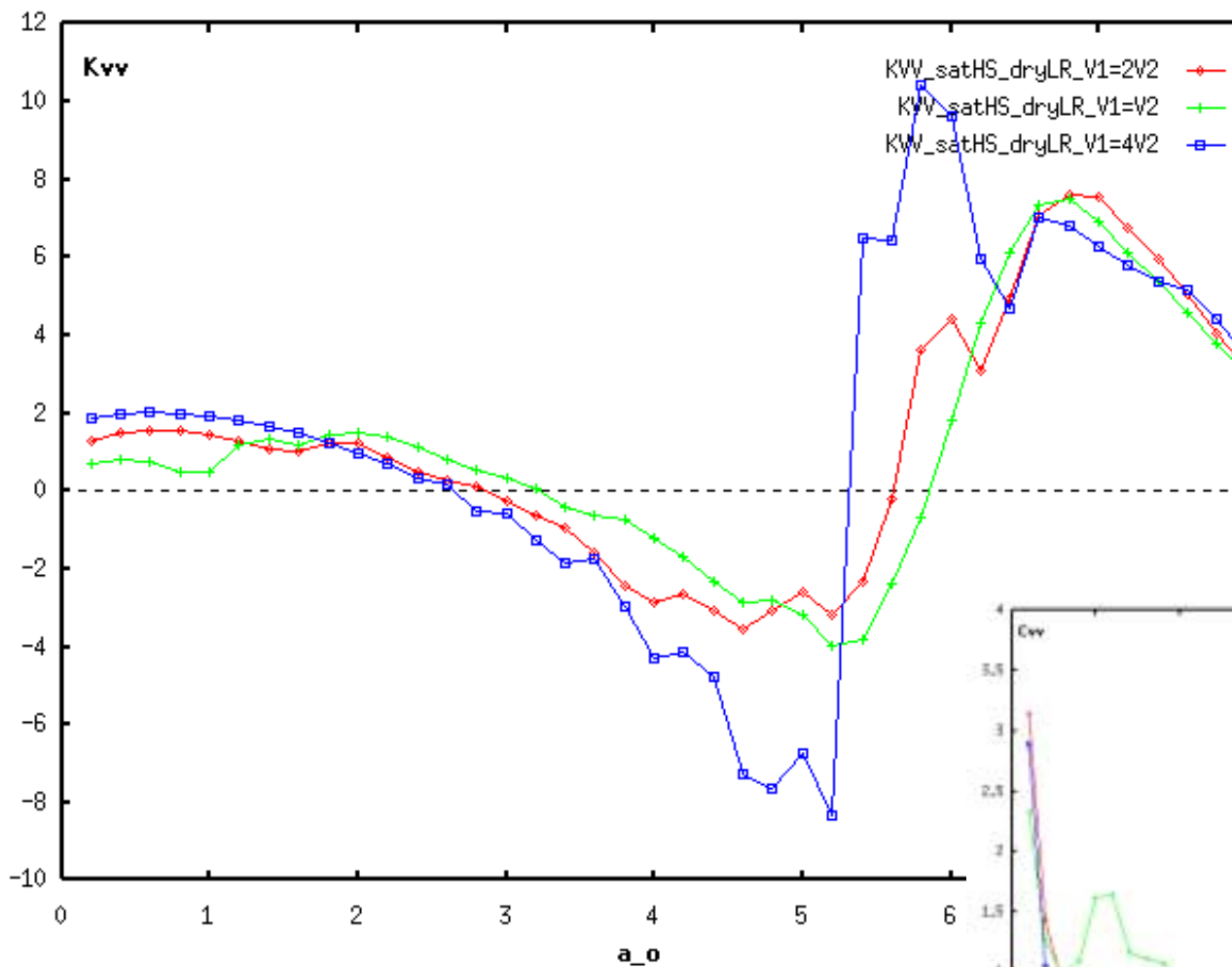
# Scattering effect of tunnel structure



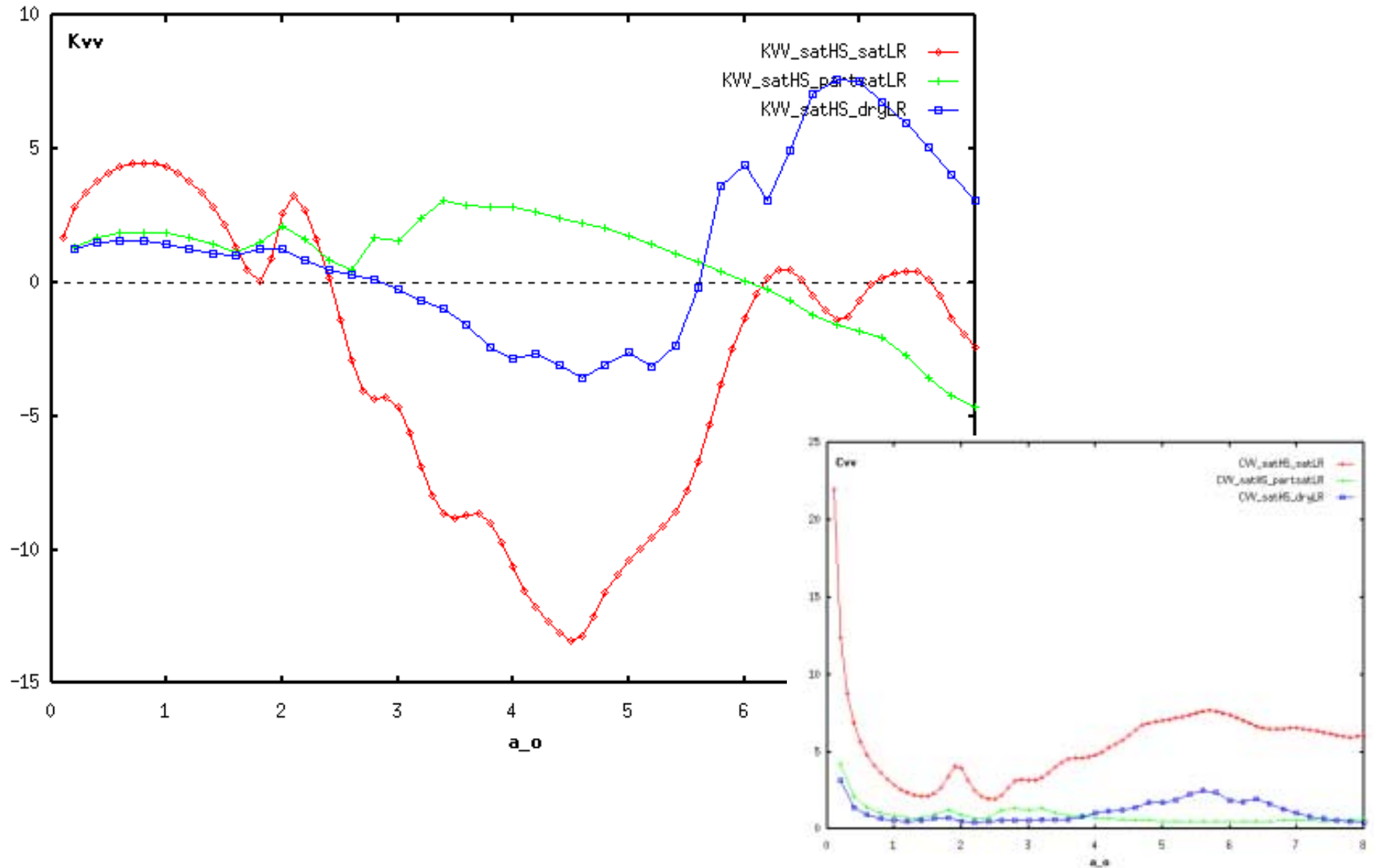
# Effect of layering – soil conditions – water table on energy dissipation



# Effect of layering – soil conditions – water table on energy dissipation



# Effect of layering – soil conditions – water table on energy dissipation



# Summary

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- Convolution and deconvolution of motion (both surface and bedrock) to the accelerator tunnel level is key to the estimate of noise induced at its floor
- The problem of solving the problem in a multi-layered soil profile is quite challenging
- Computational techniques of motion propagation and assessment of site response are available
- Knowledge of soil properties & layer depths at the site, implemented into the computing models, can help identify the best location for tunnel placement